E298A/EECS 290: Problem Set 2 Solutions

1. Plot the characteristic scattering angle as a function of voltage from 1 – 100 keV for H, C, O and Au.

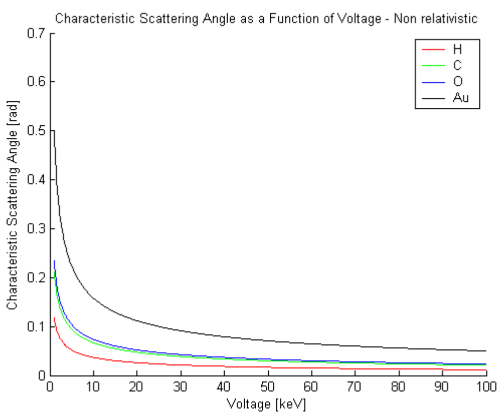
The characteristic scattering angle is given by:

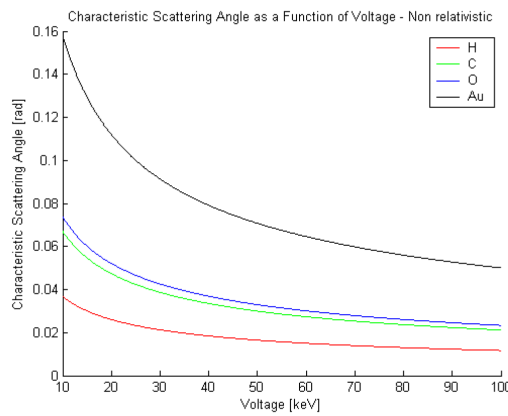
$$\theta_0 = \frac{\lambda Z^{1/3}}{2\pi a_H}$$

where $a_H = 53$ pm (Bohr radius)

$$\lambda = \frac{h}{p} = \frac{h}{\sqrt{2m_e E}}$$, ($m_e = 9.1 \times 10^{-31}$ kg, $h = 6.626 \times 10^{-34}$ J/s, E = kinetic energy)

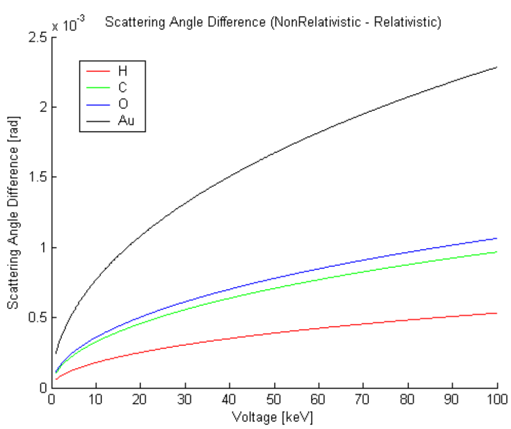
Z = atomic number

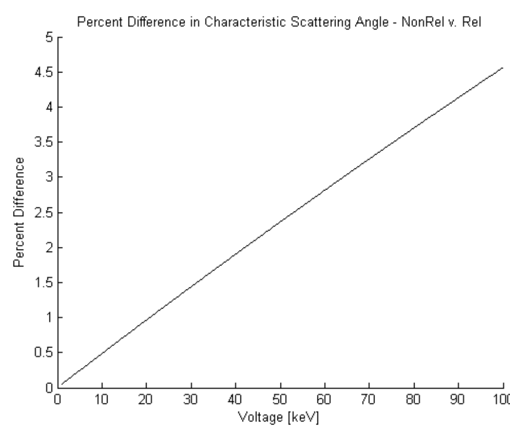




For kinetic energies around 100 keV, relativistic errors can be seen. Let us solve the previous problem using the relativistic λ and then calculate the error at high energies.

$$\lambda_{rel} = \frac{h}{p_{rel}} = \frac{h}{\sqrt{\frac{E^2}{c^2} + 2Em_e}}$$





We can see that the error in scattering angle is small and a non-relativistic approximation is still reasonable, even at 100 keV, where the error is around 5% (i.e. the non-relativistic approximation is 5% greater than the relativistic).

2. Using a weighted average of the total elastic cross sections for C, H and O, σ, calculate the elastic mean free path in a typical resist film as function of voltage. You may assume a resist density of 1.5 g/cm³ and that the atomic percentages of C, H and O are 25, 70 and 5% respectively.

Elastic cross section: $\sigma_{elastic} = \frac{Z^{4/3}\lambda^2(1+E/E_0)^2}{\pi}$

where E_0 = electron rest energy = 0.511 MeV

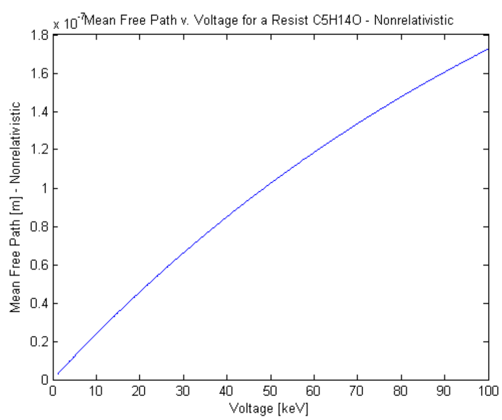
Elastic mean free path: $\Lambda = \frac{1}{N\sigma_{elastic}}$

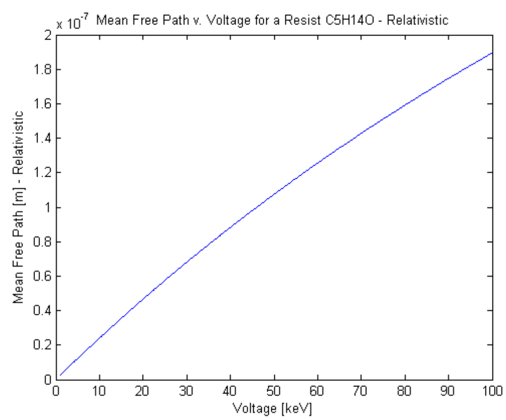
where N = atomic density = $\frac{\rho * N_A}{WeightedAtomicNumber}$

$$= \frac{\rho * N_A}{0.7A_H + 0.25A_C + 0.05A_O}$$

$$= \frac{\rho * N_A}{(0.7)+(0.25)12+(0.05)16}$$

$$\approx 2*10^{29}\,atoms/m^3$$





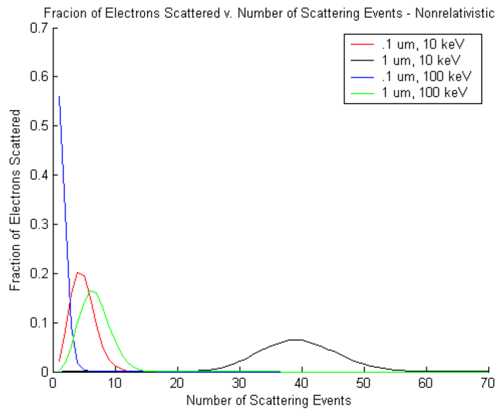
3. Calculate the probability of electrons scattered versus the number of scattering events in 0.1 um and 1 um of resist at 10 keV and 100 keV.

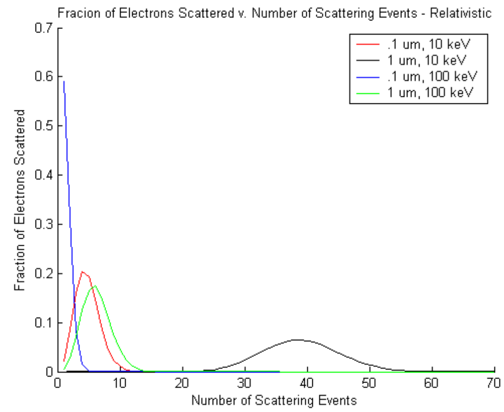
Given a particular mean free path, the probability of one electron scattered n times follows the Poisson distribution, i.e.:

$$P(n) = \frac{(t/\Lambda)^n}{n!}e^{-\frac{t}{\Lambda}}$$

From problem 2,
Nonrelativistic: E = 10 keV, $\Lambda \approx$ 26 nm, and for E = 100 keV, $\Lambda \approx$ 173 nm
Relativistic: E = 10 keV, $\Lambda \approx$ 26 nm, and for E = 100 keV, $\Lambda \approx$ 189 nm

Plotting the Poisson distribution for these parameters:





4. Calculate the beam broadening for a delta-function incident beam of 100 keV electrons in 0.1, 0.3 and 1 μm of resist. Use the mean free path, Λ, calculated in 2. Approximate the single scattering angular distribution function $S_1(\theta)$ using a Gaussian function: $S_1(\theta) = \exp(-\theta^2 / \overline{\theta_1^2})$. The mean square scattering angle, $\overline{\theta^2} = \frac{t}{\Lambda}\overline{\theta_1^2}$, is (t = thickness). Scattering in a layer between z and z + dz (z is the coordinate normal to the film thickness) increases the mean square scattering angle from $\overline{\theta^2}$ to $\overline{\theta^2} + d\overline{\theta^2}$, where $d\overline{\theta^2} = \overline{\theta_1^2}\, dz / \Lambda$. Assume a spatial broadening of the form $I(r) \propto \exp(-r \;/\, \overline{r^2})$. A scattering angle θ causes a displacement r = θz. Substitute for $\overline{\theta^2}$ and integrate over z from 0 to t. Use the weighted squares of the characteristic scattering angles, using the data from 1] and the atomic percentages of 2], to approximate $\overline{\theta_1^2}$.

The mean square scattering angle is given by:

$$\overline{\theta^2} = \frac{z}{\Lambda}\overline{\theta_1^2} \qquad (1)$$

$$\text{or } d\overline{\theta^2} = \frac{\overline{\theta_1^2}}{\Lambda} dz \qquad (2)$$

where $\overline{\theta_1^2}$ is the characteristic angle of the Gaussian approximation of the single scattering angular distribution function.

If the initial beam is a delta function, the beam diverges as it travels through the resist, at the rate described by (2). To convert the rate of change of the divergent angle into the rate of beam size broadening, note that

$$r = \theta z$$

$$\overline{r^2} = \overline{\theta^2} z^2$$

Thus,

$$d\overline{r^2} = \overline{\theta^2}\, 2z dz$$

Substituting (1),

$$d\overline{r^2} = \frac{\overline{\theta_1^2}}{\Lambda} 2z^2 dz$$

Integrating over the resist thickness *t*,

$$\overline{r^2} = \int_0^t \frac{\overline{\theta_1^2}}{\Lambda} 2z^2 dz = \frac{2}{3}\frac{\overline{\theta_1^2}}{\Lambda} t^3$$

Using non-relativistic approximations, Λ = 173 nm at 100 keV. Thus, $\sqrt{\overline{r^2}}$ can be calculated:

at 100 keV	H	C	O
θ_0 (rad)	0.0117	0.0212	0.0233

$$\overline{\theta_1^2} = 0.7*(0.0117)^2 + 0.25*(0.0212)^2 + 0.05*(0.0233)^2$$
$$= 0.000235$$

Resist thickness	$\sqrt{\overline{r^2}}$ (nm)
0.1 μm	0.95
0.3 μm	4.95
1 μm	30.13

Using relativistic calculations, Λ = 189 nm at 100 keV. Thus, $\sqrt{\overline{r^2}}$ can be calculated:

at 100 keV	H	C	O
θ_0 (rad)	0.0111	0.0202	0.0223

$$\overline{\theta_1^2} = 0.7*(0.0111)^2 + 0.25*(0.0202)^2 + 0.05*(0.0223)^2$$
$$= 0.000214$$

Resist thickness	$\sqrt{\overline{r^2}}$ (nm)
0.1 μm	0.87
0.3 μm	4.51
1 μm	27.45